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Interference and correlation of two independent photons

J. Bylander, I. Robert-Philip, and I. Abram[a]

CNRS - Laboratoire de Photonique et Nanostructures, Route de Nozay, 91460 Marcoussis, France

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Abstract. Two independent photons, produced through the spontaneous emission of two separate emitters subject to uncorrelated dephasing processes, can display two-photon interference (*i.e.* coalescence into a two-photon state) when they are incident simultaneously on a beamsplitter, in a manner analogous to that of twin photons produced through degenerate parametric fluorescence. The presence of dephasing processes, however, reduces the interference contrast (*i.e.* the probability of coalescence), by the ratio of the coherence time to the lifetime of the emitter.

PACS. 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phase measurements – 42.50.Ct Quantum description of interaction of light and matter; related experiments – 78.67.Hc Quantum dots

1 Introduction

The recent quest for photon-based quantum computing schemes has revived interest in multiphoton interference phenomena. In particular, the recent proposal for the realization of C-NOT gates and quantum computing with linear optics [1] is based on the phenomenon of photon coalescence, whereby two single photons arriving simultaneously by two different input ports on a beamsplitter, leave both by the same output port. This two-photon interference phenomenon, in which a two-photon Fock state is constructed out of two distinct one-photon Fock states, was first discovered by Hong, Ou, and Mandel (HOM) [2] who implemented the coalescence of the two twin photons emerging in a degenerate parametric fluorescence process. The HOM two-photon interferometer has proved to be an important element in the arsenal of basic quantum optics experimental techniques and was subsequently used in many studies of fundamental aspects of quantum optics, such as in experiments on the nonlocality of quantum mechanics [3] or in measuring the photon transit time in superluminal photon tunneling [4].

In all HOM experiments carried out to date, the two photons involved in coalescence had been obtained from a process of degenerate parametric downconversion, with their spectral and spatial properties being determined by appropriate spectral and spatial filtering. Twin parametric photons are automatically synchronized as they are produced simultaneously by a nonlinear process, while at the same time they are strongly correlated (and are even entangled) in frequency, wavevector, and polarization, since the nonlinear process that produces them has to obey definite conservation laws. Several theoretical papers have dealt with the coalescence of twin parametric photons [5,6].

In recent years, relatively efficient emitters have been developed that can produce single photons on demand, in view of their use in quantum cryptography and/or quantum computing. Such sources are based on the emission of a single molecule [7], a single color center [8], a single nanocrystal [9], or a single semiconductor quantum dot [10], and emit true single-photon pulses, in contrast to attenuated laser or parametric pulses whose Poisson photon statistics entail a non-negligible fraction of pulses containing two or more photons. The single-photon nature of these sources eliminates the need for post-selection procedure to weed out multiphoton occurrences, as is the case in experiments with attenuated laser or parametric pulses. At the same time, the complete absence of multiphoton pulses is an important asset both for quantum key distribution, where multiphoton occurrences can compromise security, and for any quantum information processing system where multiphoton events can reduce the visibility of the result or even alter its value.

Thus, in view of the potentialities of single photon sources in quantum information processing, it would be interesting to explore the possibility of performing two-photon interference experiments by using two single photons originating in two distinct emitters. Such a situation, however, presents an important difference from that of two twin parametric photons. The fields emerging from two different emitters have no correlation in phase, even if the two emitters are synchronized by being pumped simultaneously by a common laser pulse and are chosen to have identical transition frequencies and polarizations.

[a] e-mail: izo.abram@lpn.cnrs.fr

The lack of phase correlations arises from the coupling of the two emitters each separately to an environment that causes them to dephase independently. The dephasing of the two emitters produces, in turn, a phase diffusion of the emitted fields and this modifies the interference of the two photons and destroys partially the phenomenon of photon coalescence. The case of two independent emitters was analyzed theoretically in the past [6], but the problem of phase diffusion, which however is an inevitable feature of any physical system, was not treated.

In this paper, we revisit the theory of two-photon interference by considering explicitly the effect of dephasing processes. The paper is organized as follows: in Section 2 we describe the two ingredients entering in our model, namely, the single photon emitter subject to random phase fluctuations during emission and the propagation of the electromagnetic field in the four-port setup of a beamsplitter. In Section 3, we calculate the fourth-order photon correlation function that describes an HOM-type two-photon interferometric setup in which the two photons impinge on the two sides of the beamsplitter. These calculations serve as a guide for identifying experimental situations in which photon coalescence can be realized efficiently and can subsequently be exploited for the realization of photon-based quantum logic gates. Finally, in Section 4 we summarize our conclusions.

2 The model

We consider the situation in which two photons originating from two single-photon emitters interfere on a beamsplitter and are subsequently detected. In this section, the theoretical description of such an experiment is done in two steps: first we present the model for the material emitters and the radiative interaction subject to phase diffusion and then we deal with photon propagation in the experimental setup.

2.1 Single photons with phase diffusion

The single photon emitters are modeled each as a two-level system, with a ground state $|0\rangle_i$, and an excited state $|1\rangle_i$ of energy $\hbar\Omega$ (we assume for simplicity that both two-level systems have the same resonance frequency) and with transition operators $\hat{\pi}_i^\dagger = |1\rangle_{ii}\langle 0|$ and $\hat{\pi}_i = |0\rangle_{ii}\langle 1|$.

We further assume that each two-level system is subject to sudden, brief, and random fluctuations of its energy (arising, for example, from collisions or from interactions with thermal phonons) that are modeled by a Langevin force $F_i(t)$ representing a stationary stochastic process with delta-function correlation,

$$\langle F_i(t)\, F_j(t')\rangle = 2\Gamma\delta_{ij}\delta(t-t'), \tag{1}$$

where the angular brackets denote statistical averaging, δ_{ij} is the Kronecker delta symbol, indicating that the fluctuations of the two two-level systems are assumed to be completely uncorrelated between them, while Γ is the dephasing rate of the two-level system, which is related to the characteristic time for pure dephasing according to $T_2^* = 2/\Gamma$. For simplicity, it is assumed that both two-level systems dephase to the same rate.

The Hamiltonian for each two-level system can thus be written as,

$$\mathcal{H}_{0i} = \hbar(\Omega + F_i(t))\,\hat{\pi}_i^\dagger\hat{\pi}_i. \tag{2}$$

Each two-level system interacts with the electromagnetic field by absorbing or emitting photons whenever it undergoes a transition between its two states. The interaction Hamiltonian for each two-level system can be written as

$$\mathcal{H}_{\mathrm{I}i}(t) = g\left(\hat{A}^{(-)}(r_i,t)\hat{\pi}_i + \hat{A}^{(+)}(r_i,t)\hat{\pi}_i^\dagger\right), \tag{3}$$

where g is the radiative coupling constant and $\hat{A}^{(\pm)}(r_i,t)$ is the positive (resp. negative) frequency operator for the electromagnetic vector potential at point (r_i,t). It is essentially a sum of the annihilation (resp. creation) operators for all the modes of the electromagnetic field. The time evolution of the vector potential operators $\hat{A}^{(\pm)}(r_i,t)$ under the free field Hamiltonian

$$\mathcal{H}_{\mathrm{R}} = \frac{\epsilon_0}{2}\int_V \left(\hat{E}^2 + \hat{B}^2\right)\,\mathrm{d}^3r \tag{4}$$

corresponds to the three-dimensional diffraction of a point excitation of the electromagnetic field.

We can express the "interaction picture" Hamiltonian, for each two-level system as,

$$\begin{aligned}\tilde{\mathcal{H}}_{\mathrm{I}i}(t) &= \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\int_0^t(\mathcal{H}_{0i}+\mathcal{H}_{\mathrm{R}})\mathrm{d}t'}\mathcal{H}_{\mathrm{I}i}\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_0^t(\mathcal{H}_{0i}+\mathcal{H}_{\mathrm{R}})\mathrm{d}t'}\\ &= g\left(\hat{A}^{(-)}(t)\hat{\pi}_i\mathrm{e}^{-\mathrm{i}\hbar\Omega t-\mathrm{i}\phi_i(t)} + \hat{A}^{(+)}(t)\hat{\pi}_i\mathrm{e}^{\mathrm{i}\hbar\Omega t+\mathrm{i}\phi_i(t)}\right),\end{aligned} \tag{5}$$

where

$$\phi_i(t) = \int_0^t F_i(t')\,\mathrm{d}t' \tag{6}$$

is the fluctuating phase of the two-level system which, in the interaction picture, is transferred to the emitted photon.

We now assume that each two-level system is initially (*i.e.* at $t=0$) in its excited state $\hat{\pi}_i^\dagger|0\rangle_i$, while the field is in its vacuum state $|0\rangle_{\mathrm{Rad}}$. The time evolution under the interaction Hamiltonian (5) can be written as,

$$\begin{aligned}\hat{\pi}_i^\dagger(t)|0\rangle_i|0\rangle_{\mathrm{Rad}} = \mathrm{e}^{\frac{\mathrm{i}}{\hbar}\int_0^t\tilde{\mathcal{H}}_{\mathrm{I}i}(t')\,\mathrm{d}t'}\hat{\pi}_i^\dagger\\ \times\,\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_0^t\tilde{\mathcal{H}}_{\mathrm{I}i}(t')\,\mathrm{d}t'}|0\rangle_i|0\rangle_{\mathrm{Rad}}.\end{aligned} \tag{7}$$

It corresponds to the de-excitation of the two-level system through the emission of a photon, and can be calculated under the standard approximations of the Wigner-Weisskopf theory [11], essentially by expanding the time

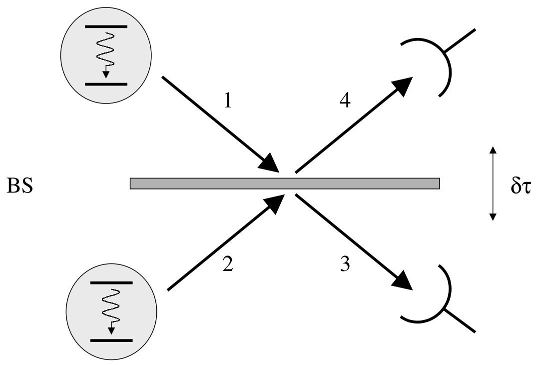

Fig. 1. Block diagram of proposed set-up for the coalescence of two independent photons on a beamsplitter (BS). The two input ports of the beamsplitter are each occupied by a two-level emitter, while the two output ports each have a single photon detector, connected to a coincidence counter. The beamsplitter can be displaced with respect to the symmetric position by a distance $\delta X = c\delta\tau$.

evolution operator up to first-order in the radiative coupling constant g and taking the long-time limit, as

$$\hat{\pi}_i^\dagger(t)|0\rangle_i|0\rangle_{\mathrm{Rad}} \to \left(\hat{\pi}_i^\dagger \mathrm{e}^{-\frac{\Gamma'}{2}t-\mathrm{i}\Delta t} - \frac{\mathrm{i}}{\hbar}g\int_0^{t\to\infty} \hat{A}^{(-)}(t') \times \mathrm{e}^{-\mathrm{i}\hbar\Omega t'-\mathrm{i}\phi_i(t')}\mathrm{e}^{-\frac{\Gamma'}{2}t'-\mathrm{i}\Delta t'}\,\mathrm{d}t'\right)|0\rangle_i|0\rangle_{\mathrm{Rad}} \quad (8)$$

where Γ' is the spontaneous emission decay rate of the two-level system, which is the inverse of the radiative lifetime $T_1 = 1/\Gamma'$, while Δ is the radiative frequency shift. In what follows, we shall concentrate on the first-order term, as it is the one that involves the emission of a photon.

2.2 Propagation

The experimental setup we consider consists of a beamsplitter with two input and two output ports, labeled $1, 2$ and $3, 4$ respectively. The ports 1 and 2 are occupied each by a single-photon emitter, and ports 3 and 4 by two photoelectric detectors that convert the single photons received into electric pulses that can subsequently be processed (Fig. 1). In general, there are four different propagation times that need to be considered, corresponding to the propagation between each emitter and each detector. In most experimental setups, three of the four propagation times are kept constant, while an adjustable delay is introduced in one of the input ports so that measurements can be made as a function of that delay. In developing the theory, however, rather than consider such an asymmetric setup with an adjustable delay in only one port, it is more convenient to consider a situation in which the emitters and the detectors are disposed symmetrically on either side of the beamsplitter, so that all four propagation times are equal (designated by τ_1), and the adjustable delay $\delta\tau$ is introduced by displacing the beamsplitter from the symmetric position [2], thus affecting simultaneously the two symmetric propagation paths that involve reflection on the beamsplitter.

In view of the experimental setup considered here, in which the optical alignment is sensitive only to rectilinear propagation along the four input/output ports of the beamsplitter, we shall approximate the full three-dimensional time evolution of the field under its Hamiltonian of equation (4) by a one-dimensional propagation along the four paths defined experimentally. We shall therefore designate by $\hat{A}_i^{(\pm)}(t - x/c)$ the vector potential operators (and by $\hat{E}_i^{(\pm)}(t - x/c)$ the electric field operators) for an excitation of the electromagnetic field (a photon) propagating along the x-coordinate of the ith port of the beamsplitter.

At the beamsplitter, the four propagating fields are related by the unitary transformation

$$\begin{aligned}\hat{E}_3^{(\pm)} &= \sqrt{T}\,\hat{E}_1^{(\pm)} + \sqrt{R}\,\hat{E}_2^{(\pm)} \\ \hat{E}_4^{(\pm)} &= \sqrt{T}\,\hat{E}_2^{(\pm)} - \sqrt{R}\,\hat{E}_1^{(\pm)},\end{aligned} \quad (9)$$

where T and R are the (intensity) transmission and reflection coefficients of the beamsplitter respectively, obeying $T + R = 1$. Taking into account that the two paths that involve transmission through the beamsplitter have a propagation time of τ_1, while the two paths that involve reflection have an additional time delay of $\delta\tau$ (positive in one case and negative in the other, as can be seen in Fig. 1), the overall transformation for the field propagating between the two emitters and the two detectors can be written as,

$$\begin{aligned}\hat{E}_3^{(\pm)}(t) &= \sqrt{T}\,\hat{E}_1^{(\pm)}(t-\tau_1) + \sqrt{R}\,\hat{E}_2^{(\pm)}(t-\tau_1+\delta\tau) \quad (10) \\ \hat{E}_4^{(\pm)}(t) &= \sqrt{T}\,\hat{E}_2^{(\pm)}(t-\tau_1) - \sqrt{R}\,\hat{E}_1^{(\pm)}(t-\tau_1-\delta\tau).\end{aligned}$$

3 Two-photon interference on a beamsplitter

The two photons, produced by the two independent two-level systems interfere on the beam-splitter, are detected by the two detectors which register, at the same time, the time of each photon arrival. The results of such an experiment are described by the $g^{(2)}$ correlation function which is of fourth order in the electric field operators $\hat{E}^{(\pm)}(t)$:

$$g^{(2)}(\tau) = \frac{\left\langle\left\langle \hat{E}_3^{(-)}(t)\hat{E}_4^{(-)}(t+\tau)\hat{E}_4^{(+)}(t+\tau)\hat{E}_3^{(+)}(t)\right\rangle\right\rangle}{\left\langle\left\langle \hat{E}_3^{(-)}(t)\hat{E}_3^{(+)}(t)\right\rangle\right\rangle\left\langle\left\langle \hat{E}_4^{(-)}(t+\tau)\hat{E}_4^{(+)}(t+\tau)\right\rangle\right\rangle} \quad (11)$$

where the double angular brackets denote quantum mechanical averaging with respect to the state of the electromagnetic field and statistical averaging with respect to the fluctuations. The numerator of equation (11) is essentially the probability of joint photodetection of one photon in port 3 at time t and one photon in port 4 at a time $t + \tau$, while the denominator permits normalization with respect to uncorrelated random events.

In accounting for propagation in space and through the beamsplitter, it is possible to propagate either the state of the electromagnetic field or the operators that describe the detection process. In this paper, we take the latter approach as it gives expressions that are more compact, by using the transformation (10) in which we incorporate for simplicity the propagation time τ_1 in the definition of t. Thus, using the time evolution of the two-level systems (8) that describes photon emission, the "ket" of the numerator in equation (11) can be written as:

$$\int_0^\infty \mathrm{d}t' \int_0^\infty \mathrm{d}t'' \Big(T\,\hat{E}_2^{(+)}(t+\tau)\hat{E}_1^{(+)}(t) \\ -R\,\hat{E}_1^{(+)}(t+\tau-\delta\tau)\hat{E}_2^{(+)}(t+\delta\tau)\Big)\,\hat{A}_1^{(-)}(t')\hat{A}_2^{(-)}(t'') \\ \times\,\mathrm{e}^{-\mathrm{i}\phi_1(t')-\mathrm{i}\phi_2(t'')-\mathrm{i}(\Omega+\Delta)(t'+t'')}\mathrm{e}^{-\frac{\Gamma'}{2}(t'+t'')}|0\rangle_{\mathrm{Rad}} \quad (12)$$

where the cross-terms in $\sqrt{RT}$ were not retained, because the corresponding detection process involves the annihilation of two photons both in the same mode (1 or 2). Thus, these cross-terms vanish when applied to a state that contains one photon in each one of the two modes.

The "ket" of equation (12) can be readily evaluated by commuting the two electric field (annihilation) operators through the two vector potential (creation) operators. This can be done by using the canonical commutator for the electromagnetic field,

$$\Big[\hat{E}_i^{(+)}(r,t),\,\hat{A}_j^{(-)}(r',t')\Big] = \frac{\mathrm{i}\hbar}{\epsilon_0}\,\delta_{ij}\delta^{\perp}(r-r'-ct+ct') \quad (13)$$

where $\delta^{\perp}(x)$ is the transverse delta function which, in our simplified linear geometry that involves only transverse fields, reduces to the ordinary delta function. Operating this commutation, the only surviving terms are those proportional to the commutator, so that the "ket" can be evaluated as,

$$\int_0^\infty \mathrm{d}t' \int_0^\infty \mathrm{d}t''[T\,\delta(t+\tau-t'')\delta(t-t') \\ -R\,\delta(t+\tau-\delta\tau-t')\delta(t+\delta\tau-t'')] \\ \times\,\mathrm{e}^{-\mathrm{i}\phi_1(t')-\mathrm{i}\phi_2(t'')-\mathrm{i}(\Omega+\Delta)(t'+t'')}\mathrm{e}^{-\frac{\Gamma'}{2}(t'+t'')}|0\rangle_{\mathrm{Rad}}. \quad (14)$$

The delta functions will give a contribution to the integrals only when their arguments are in the positive time axis and this ensures that the decay terms involve only positive times. Upon integration of the delta functions, the "ket" thus becomes

$$\Big[T\,\mathrm{e}^{-\mathrm{i}\phi_1(t)-\mathrm{i}\phi_2(t+\tau)}\mathrm{e}^{-\frac{\Gamma'}{2}|\tau|} \\ -R\,\mathrm{e}^{-\mathrm{i}\phi_1(t+\tau-\delta\tau)-\mathrm{i}\phi_2(t+\delta\tau)}\mathrm{e}^{-\frac{\Gamma'}{2}|\tau-2\delta\tau|}\Big] \\ \times\,\mathrm{e}^{-\mathrm{i}(\Omega+\Delta)(2t+\tau)}\mathrm{e}^{-\Gamma' t}|0\rangle_{\mathrm{Rad}}. \quad (15)$$

The "bra" part of the numerator, which involves the negative frequency photodetection operators, is the Hermitian conjugate of the "ket" (12), so that the overall expectation value in the numerator of equation (11) is,

$$\Big[T^2\mathrm{e}^{-\Gamma'|\tau|} + R^2\mathrm{e}^{-\Gamma'|\tau-2\delta\tau|} \\ -RT\,\Big(\Big\langle \mathrm{e}^{\mathrm{i}\phi_1(t)+\mathrm{i}\phi_2(t+\tau)-\mathrm{i}\phi_1(t+\tau-\delta\tau)-\mathrm{i}\phi_2(t+\delta\tau)}\Big\rangle + \mathrm{c.c.}\Big) \\ \times\,\mathrm{e}^{-\frac{\Gamma'}{2}|\tau|-\frac{\Gamma'}{2}|\tau-2\delta\tau|}\Big]\mathrm{e}^{-2\Gamma' t}, \quad (16)$$

where "c.c." denotes the complex conjugate, while the angular brackets denote statistical averaging. Thus, averaging over the fluctuating phase as explained in the Appendix, the numerator gives

$$\Big[T^2\mathrm{e}^{-\Gamma'|\tau|} + R^2\mathrm{e}^{-\Gamma'|\tau-2\delta\tau|} \\ -2RT\mathrm{e}^{-\Gamma|\tau-\delta\tau|}\mathrm{e}^{-\frac{\Gamma'}{2}|\tau|-\frac{\Gamma'}{2}|\tau-2\delta\tau|}\Big]\mathrm{e}^{-2\Gamma' t}. \quad (17)$$

In practice, a coincidence experiment is done with relatively slow detectors and the temporal resolution is obtained essentially from the delay $\delta\tau$ introduced in the beam paths. Under such conditions the two temporal variables related to the detectors, t and τ in equation (17), should the integrated over the resolving time of the detectors, which can be taken to be infinite, since it is usually much longer than any of the characteristic times of the experiment, such as the coherence time or the excited state lifetime. Upon integration and normalization we therefore obtain the dependence of the second-order correlation function on the time delay $\delta\tau$ as,

$$g^{(2)}(\delta\tau) = 1 - \frac{2RT}{1-2RT}\left[\frac{\Gamma'}{\Gamma+\Gamma'}\mathrm{e}^{-(\Gamma+\Gamma')|\delta\tau|} \right. \\ \left. +\frac{\Gamma'}{\Gamma}\left(\mathrm{e}^{-\Gamma'|\delta\tau|} - \mathrm{e}^{-(\Gamma+\Gamma')|\delta\tau|}\right)\right]\cdot \quad (18)$$

This equation can also be expressed in terms of the characteristic times of the two-level systems, that is, the excited state lifetime T_1, the pure dephasing time T_2^* and the coherence time T_2 defined by

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_2^*} \quad (19)$$

as

$$g^{(2)}(\delta\tau) = 1 - \frac{2RT}{1-2RT}\left[\frac{T_2}{2T_1}\mathrm{e}^{-2|\delta\tau|/T_2} \right. \\ \left. +\frac{T_2^*}{2T_1}\left(\mathrm{e}^{-|\delta\tau|/T_1} - \mathrm{e}^{-2|\delta\tau|/T_2}\right)\right]\cdot \quad (20)$$

It should be noted that the coherence time T_2 corresponds to the reciprocal of the linewidth of the emission spectrum of the two-level system.

For a 50–50 beamsplitter $T = R = 1/2$, so that at zero time delay, $\delta\tau = 0$, the joint photodetection probability is

$$g^{(2)}(0) = \frac{\Gamma}{\Gamma+\Gamma'} = 1 - \frac{T_2}{2T_1}\cdot \quad (21)$$

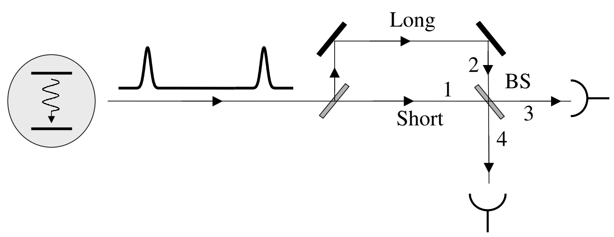

Fig. 2. Unbalanced Mach-Zehnder interferometer for observing the coalescence of two photons emitted by a single quantum dot, following two consecutive excitation pulses separated by an interval Δt. When the first photon takes the long path and the second photon takes the short path, they both arrive simultaneously on the beamsplitter (BS) and can coalesce. In all other path configurations the two photons are separated by an interval of Δt or $2\Delta t$ so that these events can be discriminated by the coincidence electronics.

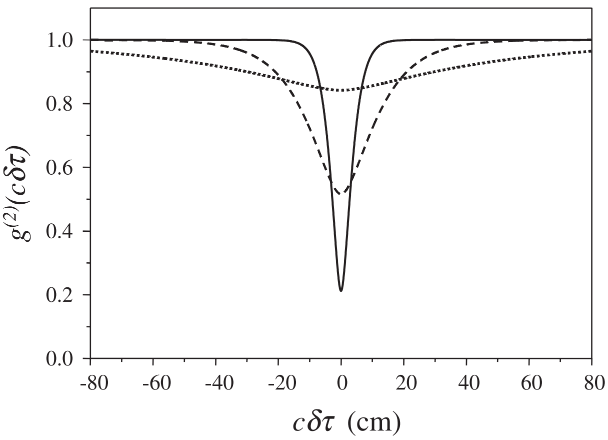

Fig. 3. Coincidence probability as a function of beamsplitter displacement, $g^{(2)}(c\delta\tau)$, for two photons emitted by two independent two-level systems displaying a pure dephasing time of 600 ps, for three different values of the excited state lifetime. (a) Dotted line: $T_1 = 1.6$ ns, corresponding to the case of bare InAs quantum dots, (b) dashed line: $T_1 = 320$ ps, corresponding to InAs quantum dots with a lifetime enhancement of a factor of 5 by cavity effects, (c) full line: $T_1 = 80$ ps, corresponding to InAs quantum dots with a lifetime enhancement of a factor of 20 by cavity effects.

That is, the second-order correlation function corresponds to a curve that at long time delays takes the value of 1, while at zero time delay it displays a dip whose depth is given by equation (21) and whose width is on the order of T_2.

In the absence of pure dephasing processes in the two-level systems, when the emitted photons are "Fourier transform limited", (that is their temporal profile corresponds to the Fourier transform of their spectrum) we have $T_2 = 2T_1$ and the central dip goes all the way down to 0 and its width coincides with the duration of the photon wavepackets. This means that, in the absence of dephasing, every time the photons arrive simultaneously on the beamsplitter they coalesce into a two-photon state and both leave by the same output port, as in the case of parametric photons. However, in the presence of dephasing, the depth and the width of the coalescence dip are reduced, which means that the photons will coalesce only if they arrive within a time interval given by their coherence time T_2, and even in that case the efficiency of photon coalescence is reduced to

$$P = \frac{T_2}{2T_1} \cdot \tag{22}$$

An alternate interpretation of the reduction of the coalescence dip rests on the visualization of the spontaneous emission process in the presence of dephasing as consisting of the emission of photon wavepackets of duration $T_2/2$, giving rise to a dip with the corresponding width. However the emission of the photons occurs randomly within the excited state lifetime and thus involves a time jitter of the order of T_1. The probability that two randomly emitted photon wavepackets will overlap is given by equation (22), hence the reduced depth of the dip.

In order to illustrate our results with a concrete example, we shall consider the case of semiconductor quantum dots which are at present the most promising system for producing transform-limited single photons. As it is difficult to obtain two quantum dots with identical frequencies to place in the two input ports of the beamsplitter, an alternative experimental situation can be envisaged as follows: a single dot, excited with two consecutive laser pulses separated by a time interval $\Delta t \gg T_1$, emits two single photon pulses separated by the same time interval Δt. These two consecutive emissions are characterized by the same parameters T_1 and T_2 but are completely uncorrelated. The two photons are then sent through an unbalanced Mach-Zehnder interferometer (see Fig. 2) in which one arm introduces a delay of Δt, so that when the first photon follows the long arm and the second photon the short arm, the two photons arrive simultaneously on the exit beamsplitter of the interferometer, and photon coalescence can take place. The events in which the photons follow a different configuration of paths in the interferometer produce delays of Δt or $2\Delta t$ between the photons, so that these events can be easily discriminated if the time interval Δt is long enough to be resolved by the coincidence processing electronics. Figure 3 is a plot of equation (20) for three different cases involving InAs quantum dots. The characteristic times of InAs quantum dots are typically on the order of $T_1 = 1.6$ ns [12] and $T_2^* = 600$ ps [13], (dotted curve in Fig. 3) which implies that in an experimental setup involving bare InAs quantum dots, the coalescence efficiency is only $P \approx 0.19$, a value which is too low for any practical implementation in quantum information processing. Improvement of this efficiency can be achieved by reducing the radiative lifetime of the quantum dot through its introduction in a resonant microcavity and the exploitation of the Purcell effect [14]. A Purcell effect enhancing the spontaneous emission by a factor of $F_p = 5$ (dashed curve) gives a radiative lifetime of $T_1 = 320$ ps and a coherence time of $T_2 = 305$ ps producing a coalescence efficiency of $P \approx 0.49$. A coalescence efficiency of $P \approx 0.79$, a value which is high enough to envisage applications in quantum information processing, can be obtained if the spontaneous emission of the quantum dots is enhanced by a Purcell factor on the order of 20 (solid curve) which produces a radiative lifetime of $T_1 = 80$ ps and a coherence time of approximately $T_2 = 126$ ps. Such Purcell factors have already been achieved [15], indicating that the possibility of realizing photon coalescence with semiconductor quantum dots embedded in pillar microcavities is totally accessible with available technology.

It should be noted, that in order for photon coalescence to be a convincingly quantum effect, it has to have an efficiency of at least 50%. Indeed, an experiment [16] involving a quasi-classical beam of macroscopic intensity split into two and then re-combined on a beamsplitter in a manner analogous to that of the HOM setup, displays a dip in the $g^{(2)}$ correlation function going down to $g^{(2)} = 1/2$.

4 Summary and conclusions

We have studied the interference of two photons emitted by two independent two-level systems that undergo dephasing in the course of the emission process. When the two photons are combined on a beamsplitter, they both exit the beamsplitter through the same output port, coalescing into a two-photon state, whereas in the absence of interference the photon trajectories are uncorrelated and the two photons can leave the beamsplitter separately by the two output ports. Such an experiment can be described through the $g^{(2)}$ correlation function which gives the probability of observing a photon in one output port at time t and a photon at time $t + \tau$ in the other output port. If interference occurs and the two photons coalesce, the probability for the two photons leaving separately is zero so that $g^{(2)} = 0$, whereas for non-interfering photons impinging on the beamsplitter, there is a 50% probability for detecting a photon in each output port so that $g^{(2)} = 1$.

We have calculated the $g^{(2)}$ correlation function for the interference of two photons originating in two emitters characterized by an excited state lifetime T_1 and a coherence time T_2, by modeling the dephasing process as resulting from stochastic fluctuations of the excited state energy of the emitter corresponding, for example, to fluctuations due to collisions. Our results indicate that interference does occur also with independent photons, when the two photons arrive simultaneously on the beamsplitter within a time interval of the order of the coherence time T_2 (given by the reciprocal of the spectral width) as in the case of interference of twin degenerate parametric photons. However, unlike the interference of parametric photons, the $g^{(2)}$ function for interference of independent photons does not drop to zero, but only to $1 - T_2/(2T_1)$. This can be interpreted by viewing the emission process in the presence of dephasing as involving a statistical uncertainty in the time of photon emission of the order of $T_2/(2T_1)$. This time jitter reduces the probability that the two photons arrive on the beamsplitter simultaneously and spoils the efficiency of photon coalescence by the same factor.

Calculations using the physical parameters of InAs quantum dots, one of the leading single-photon sources at present, indicate that, because of the relatively rapid dephasing, the efficiency of coalescence of photons produced by bare InAs quantum dots is relatively low (of the order of 20%). However, by enhancing the spontaneous emission lifetime of the quantum dots by a factor of 20, by use of Cavity Quantum Electrodynamics effects, it is possible to raise this efficiency to levels that could become interesting for quantum information processing schemes (of the order of 80%). Experiments are in progress in our laboratory to implement these ideas.

Numerous helpful discussions with R. Ghosh, Ph. Grangier, and J.-Ph. Poizat are gratefully acknowledged.

Appendix A: Statistics of dephasing fluctuations

In this appendix, we review the Langevin theory for collisional broadening of photon emission. We assume that the excited state energy of the emitter is subject to random fluctuations described by a stochastic stationary Langevin force of zero mean

$$\langle F(t)\rangle = 0 \tag{A.1}$$

and delta function correlation

$$\langle F(t)\, F(t')\rangle = 2\Gamma\delta(t - t'), \tag{A.2}$$

where the angular brackets denote statistical averaging.

As the excited state evolves in time, it will acquire phase given by

$$\phi(t) = \int_0^t F(\tau)\,\mathrm{d}\tau \tag{A.3}$$

which is also a random variable with zero mean

$$\langle\phi(t)\rangle = \int_0^t \langle F(\tau)\rangle\,\mathrm{d}\tau = 0, \tag{A.4}$$

while its two-time correlation function can be calculated as,

$$\begin{aligned}\langle\phi(t_1)\,\phi(t_2)\rangle &= \int_0^{t_1}\mathrm{d}\tau_1\int_0^{t_2}\mathrm{d}\tau_2\langle F(\tau_1)\,F(\tau_2)\rangle \\ &= \Gamma\times(t_1 + t_2 - |t_1 - t_2|).\end{aligned} \tag{A.5}$$

All higher order correlation functions of odd order are equal to zero, while the correlation functions of even order break up into all the possible combinations of products of second order correlation functions.

In calculating the average of the electric field being emitted, we usually have to deal with exponentials of the phase. The statistical averaging can then be performed by expanding the exponential into a power series and then deal with all the combinations of second-order correlation functions that result from that expansion. In particular, we have

$$\langle e^{i\phi(t)} \rangle = e^{-\Gamma t} \to 0, \quad (A.6)$$

where we have taken the stationary limit $\Gamma t \to \infty$. Similarly,

$$\langle e^{i\phi(t_1)-i\phi(t_2)} \rangle = e^{-\Gamma|t_1-t_2|}, \quad (A.7)$$

giving rise to the familiar exponential decay of the coherent amplitude due to dephasing.

When several emitters are present, we assume that their fluctuations are uncorrelated so that the statistical averages are performed separately for each emitter.

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